

A FAST COMPUTATIONAL ALGORITHM FOR TRANSIENT STABILITY  
ASSESSMENT OF POWER SYSTEMS

طريقة سريعة لتقدير الاستقرار العابر لنظم القوى

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ملخص: تستخدم طريقة دالة الطاقة العابرة في تحديد زمن القصر المخرج للنظام الكهربائي. في هذه الطريقة تحسب الطاقة المخرجة عن طريق معدل التغير في طاقة الحركة. يستخدم مفكوك تيلور للتنبؤ بقيمة سرعة و زاوية العنصر الدوار لكل من المولدات وذلك لسرعة حساب زمن القصر المخرج.

زمن القصر المخرج السابق حسابه يستخدم في اختيار الضبط الزمني الملائم لمرحلات الوقاية الثانوية لتجنب عدم استقرار النظام او الفصل الخاطيء لمرحلات الوقاية.

اللو جاريتم المقترح طبق على نظامين. استخدم النظام الأول للتأكد من دقة البرنامج ومقارنة النتائج، بينما النظام الثاني يمثل منطقة الدلتا من الشبكة الموحدة لجمهورية مصر العربية.

#### Abstract

The transient energy function method is used to determine the critical clearing time of a power system. In this method, the critical energy is determined in terms of the rate of change of kinetic energy. For fast computation of critical clearing time, angles and speeds of system generators are predicted through Taylor series expansion.

Early computed critical clearing time is used for choosing the proper failure time of backup protection, to prevent both of system instability and false tripping of protective relays.

The proposed algorithm is applied on two power systems. The first system is used for comparing results, while the second represents Delta region of unified power system of Egypt.

#### 1. INTRODUCTION

Considerable progress has been made in power system transient stability analysis using transient energy function (TEF) method [1,2,3]. The transient energy function (V) consists of two components: kinetic energy ( $V_{KE}$ ) and potential energy ( $V_{PE}$ ). When the power system's equilibrium is disturbed, there is an excess (or deficiency) of energy associated with synchronous machines, setting them to move or swing away

from equilibrium. The power system ability to absorb excess energy depends largely on the post-fault system configuration [2]

The basic idea of the TEF method is to compare two values of transient energy. One is called critical value ( $V_{cr}$ ) which is the potential energy at the unstable equilibrium point (UEP). The other value is the energy at the end of disturbance. The time at which the value of TEF is equal to  $V_{cr}$ , is the critical clearing time (CCT). Comparing with the method of digital simulation of system dynamics, the TEF method has a short consuming time for determining the CCT [1,3].

Several methods have been proposed to determine the critical energy. One method is based on computation of TEF at a particular UEP [1,2]. This requires knowledge of generators rotor angles (with respect to COA reference frame) for all the UEP's. Computationally, this is difficult and the iterative methods used in computation may have convergence problems. Another method is based on the maximum potential energy. In which, the critical energy may be equal to the maximum potential energy component of the TEF along the faulted trajectory. The drawback of this method is the assumption of constant machine acceleration [7,9].

Rate of change of kinetic energy (RCKE) method is proposed for CCT computation. [3]. In this method, the CCT is the time at which the RCKE of individual machines reaches its maximum negative value. This method assumes that, for each machine, the system separates into one-machine-infinite-bus (OMIB) system. Calculating the OMIB parameters, for each machine, may take a long time compared to system transient time.

This paper proposes a method for determining the critical clearing time of the power system, following a large disturbance, using the rate of change of kinetic energy of the system. The critical energy is estimated without determining the UEP. Fast evaluation of critical clearing time can be achieved through Taylor series expansion, which is used to predict the system states (generator's rotor angles and speed deviations). The estimated CCT is used for determining the appropriate failure time and coordinating margin factor of backup protection.

## 2. THE POWER SYSTEM MODEL

The power system is represented by a classical generator model, where the voltage behind  $X_d'$  and mechanical input are constants. The dynamics of each machine, in center of angle (COA) reference frame, are represented by the following equations [1]:

$$M_i \dot{\omega}_i = P_i - P_{ei} - (M_i/M_T) P_{COA} \quad (1)$$

$$\dot{\theta}_i = \omega_i \quad i=1, 2, \dots, n$$

The expressions for  $P_i, P_{ei}, P_{COA}$  and  $M_T$ , respectively, are given by

$$P_i = P_{mi} - E_i^2 G_{i,i}$$

$$P_{ei} = \sum_{\substack{j=1 \\ j \neq i}}^n [C_{ij} \sin \theta_{ij} + D_{ij} \cos \theta_{ij}]$$

$$P_{COA} = \sum_{i=1}^n P_i - 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n D_{ij} \cos \theta_{ij} \quad (2)$$

$$M_T = \sum_{i=1}^n M_i$$

where, for generator  $i$

- $\theta_i, \omega_i$  = generator's angular position and angular velocity deviation with respect to COA respectively
- $M_i$  = generator inertia constant
- $E_i \angle \delta_i$  = voltage behind transient reactance
- $P_{mi}$  = mechanical power input
- $C_{ij} = E_i E_j B_{ij}$
- $D_{ij} = E_i E_j G_{ij}$
- $G_{ij}, G_{ij}, B_{ij}$  are obtained from the network's reduced Y-matrix

## 2.1 The Transient Energy Function

Associated with eqn. (1) the energy function  $V$  describing the total system transient energy is always defined for the post-disturbance of the system, is given by [2]:

$$V = \frac{1}{2} \sum_{i=1}^n M_i \omega_i^2 - \sum_{i=1}^n P_i (\theta_i - \theta_i^s) - \sum_{i=1}^{n-1} \sum_{j=i+1}^n [C_{ij} (\cos \theta_{ij} - \cos \theta_{ij}^s) - D_{ij} \frac{\theta_i + \theta_j - \theta_i^s - \theta_j^s}{\theta_i - \theta_j - \theta_i^s + \theta_j^s} (\sin \theta_{ij} - \sin \theta_{ij}^s)] \quad (3)$$

where  $\theta_{ij}^s$  ( $i=1 \sim n$ ) is the angle at stable equilibrium point (SEP). The first term in eqn. (3) represents kinetic energy and the remainder of the terms constitute the potential energy of the system. The last term

is a linear trajectory approximation for the path dependent integral component of the transfer conductance [2].

## 2.2 Prediction of System States Through Taylor Series Expansion

The Taylor series expansions for generator rotor angles and angular velocity deviation can be expressed as follows [8]:

$$\begin{aligned} \theta_i(t) &= \theta_i(0) + \theta_i^{(1)} t + \theta_i^{(2)} \frac{t^2}{2!} + \theta_i^{(3)} \frac{t^3}{3!} + \dots \\ \omega_i(t) &= \omega_i(0) + \omega_i^{(1)} t + \omega_i^{(2)} \frac{t^2}{2!} + \omega_i^{(3)} \frac{t^3}{3!} + \dots \end{aligned} \quad (4-a)$$

where  $\theta_i(0)$  = the prefault angle of generator  $i$  with respect to COA  
 $\theta_i^{(n)}$  = the  $n$ th derivative of  $\theta_i$  at  $t=0$

At  $t \geq t_{cl}$ , the post-fault network parameters are used in eqn. (4-a). In this case:

$$\begin{aligned} \theta_i(0) &= \theta_i(t_{cl}) \\ \omega_i^{(1)} &= \omega_i(t_{cl}) \end{aligned} \quad (4-b)$$

and other derivatives are calculated at  $t_{cl}$ . The expressions for the coefficients of faulted, as well as, post-fault system are given in Appendix 1.

Eqn.(4) predicts the system states accurately for a duration time of 0.4~0.5 s, considering terms up to 4th order derivatives [4]. However, if prediction of system states for longer duration is desired, a second or multistep TSE in both the fault, as well as, post-fault periods may be used, if necessary.

## 3. RATE OF CHANGE OF KINETIC ENERGY (RCKE)

From eqn.(3), the system kinetic energy is

$$V_{KE} = \frac{1}{2} \sum_{i=1}^n M_i \omega_i^2 \quad (5)$$

Differentiating eqn. (5) with respect to time, the RCKE of the system is

$$\frac{dV_{KE}}{dt} = \sum_{i=1}^n \left[ M_i \omega_i \left( \frac{d\omega_i}{dt} \right) \right] \quad (6)$$



Substituting eqn.(1) into eqn.(6) gives

$$RCKE = \sum_{i=1}^n \omega_i [ P_i - P_{oi} - (M_i/M_T) P_{COA} ] \quad (7)$$

This RCKE explains how the system disturbing kinetic energy is absorbed and converted into potential form. In disturbed system, more disturbed generators may or may not lose synchronism with the rest of the system, depending on whether the potential energy absorbing capacity of the network, is adequate to convert the transient kinetic energy of the system at the end of the disturbance into potential energy or not.

Since the total energy ( $t \geq t_{cl}$ ) is constant, it is clear that

$$RCKE = dV_{KE}/dt = - dV_{PE}/dt \quad (8)$$

Hence  $V_{PE}$  can be obtained by integrating eqn. (8) with respect to time. This gives

$$V_{PE} = - \int_0^{t_{cl}} (RCKE) dt \quad (9)$$

R.H.S of eqn. (9) represents the transient VKE at the end of disturbance. If the fault is kept long enough, the injected kinetic energy of the system reaches to its maximum value, when the RCKE value equals zero. Assuming  $t_m$  is the corresponding clearing time (Fig.3).

After the removal of the disturbance, no new transient VKE is injected into the system. Thus the critical energy  $V_{cr}$  should be equal to the R.H.S of eqn.(9) at the clearing time  $t_m$  as follows

$$V_{cr} = - \int_0^{t_m} (RCKE) dt \quad (10)$$

### 3.1 An Algorithm for Computation of CCT

The algorithm for computing the critical clearing time by the RCKE method may be presented as follows:

- Step 1: using the prefault load flow data, compute the initial state
- Step 2: obtain the Taylor series expansions of eqn. (4) for the faulted rotor angle and speed deviation
- Step 3: select a clearing time  $t_{cc}$ , Starting with a reasonably small value. Compute RCKE of the system, using eqn.(7), at the instant of fault clearance
- Step 4: If  $RCKE \neq 0$  increase the clearing time by a suitable increment and repeat step 3
- Step 5: If  $RCKE = 0$ , record the clearing time. This time is  $t_m$ , at which the VKE reaches its maximum value

Step 6: calculate the critical energy  $V_{cr}$ , using eqn. (10)  
 Step 7: calculate the total energy  $V$  from eqn. (3) and compare it with  $V_{cr}$ , obtained from step 6. The time at which  $V = V_{cr}$  is the critical clearing time.

#### 4. RESULTS OF STUDY

For the purpose of comparison, the CCTs are calculated using the classical step by step method, and the RCKE method. The results obtained are as follows:

##### 4.1 Three- machine, nine- busbar system (system #1) [5]

The system is shown in Fig.1. A 3-phase to ground fault is applied near busbar 7. It is cleared at 0.16 s. Circuit breakers at both ends of line 5-7 are opened for fault clearance.

Fig.2 shows the generators swing curves by simulation and by application of a single step TSE of eqn.(4). For time duration of 0.4 s, the curves obtained by the two methods coincide. But, for longer duration, the multistep TSE is required.

Fig.3 shows the RCKE and the VKE of the system against the clearing time. It is shown that at  $t_m = 0.16$  s, the RCKE passes through zero and the area under the curve reaches its maximum value.

The variation of VKE, VPE, and system energy  $V$  are also shown in Fig.4. The critical clearing time CCT is the time at which  $V_{cr}$  equals  $V$ , as shown in Fig.4. In this case, CCT is 0.20 s.

The estimated CCTs using the proposed method and using digital simulation for three faults studied in this system are given in Table 1.

Table 1.

Fault location Busbar no.	line tripped between buses	Clearing time $t_m$ , s for RCKE = 0	CTT(s) obtained by	
			proposed method	digital simulation
7	5-7	0.16	0.20	0.19
9	6-9	0.18	0.22	0.22
4	4-6	0.28	0.32	0.30

##### 4.2 Five-machine, fifteen-busbar typical power system (system #2)

This system represents the reduced power network of Delta region of unified power system of Egypt. The system single line diagram is shown in

Fig.5. Its data and initial operating conditions are given in Appendix 2. These data are actual as recorded in Egypt national load dispatch center [6]. The total load of 1711 MW recorded on 15/6/1993, is used in the study.

Fig.6 shows the generator angles response for 3-phase fault at busbar 11, with duration of 0.12 s. The fault is cleared by opening one of the two lines between busbars 11 and 9.

Fig.7 shows the comparison between simulation method and single step TSE, when they are used in computation of generator swing curves. The results are coinciding in the first period of 0.4 s.

For the above fault location, the critical energy is computed and compared with the total energy of the system, as shown in Fig.8. The critical clearing time in this case is 0.42 s.

Faults at three different locations are considered in the system. Table 2 shows the estimated CCTs obtained by the proposed method.

Table 2.

Fault location B.B no.	line tripped between buses	CCT obtained by proposed method (s)
9	9-11	0.38
11	9-11	0.42
13	13-21	0.57

#### 4.3 Case study on a local circuit breaker failure protection

The above computed CCT can be used for determining the limits of CBF clearing time and coordinating margin factor, for each area of the power system.

The primary objective of a CBF protection scheme (Fig.9) is to open all sources to an uncleared fault on a system. To do this efficiently, the CBF scheme must operate fast enough to maintain system stability, prevent excessive equipment damage and maintain a prescribed degree of service continuity [10]. This leads to the necessity of coordinating the CBF relays with other relays on the system. Fig.10 shows a flow chart of all of these related factors.

The problem of choosing the proper failure time is extremely complex, involving the effect on the system stability, the type of relays and the circuit breakers.

The proposed algorithm may be used to evaluate the CCT of a three phase fault at each bus in the grid. The proper setting of the backup and CBF protection, based on the evaluated CCT and the installed protection gears operation time, can be obtained using Fig.10.

## 5. CONCLUSIONS

A method of estimating critical clearing time for a disturbed multimachine power system is proposed. For a certain fault location, the fault clearing time is increased until the injected kinetic energy of the system reaches its maximum value. This occurs when rate of change of kinetic energy passes through zero. The rate of change of kinetic energy of the system is used to evaluate the system critical energy. The critical clearing time is the time at which the whole system energy equals the critical energy.

Calculation of critical energy by the proposed method is simple. It requires calculation of few coefficients, which depend on quantities such as angle, speed, power, etc. of the post faulted system. These quantities can be fast predicted using Taylor series expansion.

The proposed method is fast in terms of computation, compared with one of the existing methods of calculating clearing times (simulation of system dynamics).

The proposed method can be used as a tool for determining the failure time setting of backup protection and CBF where quick determination of critical clearing time is of interest.

## 6. REFERENCES

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## APPENDIX 1

Coefficients of Taylor series expansion [4]

The second and the higher derivatives of  $\theta$  can be derived by differentiating eqn. (1) with respect to time. These derivatives are

$$\begin{aligned}
 \theta_i^{(2)} &= \frac{P_{mi}}{M_i} - \sum_{j=1}^n A_{ij} + d_1 \\
 \theta_i^{(3)} &= \sum_{j=1}^n B_{ij} \theta_{ij}^{(1)} + d_2 \\
 \theta_i^{(4)} &= \sum_{j=1}^n [A_{ij} \theta_{ij}^{(1)2} + B_{ij} \theta_{ij}^{(2)}] + d_3
 \end{aligned} \tag{A1}$$

The parameters  $d_1$ ,  $d_2$  and  $d_3$  are determined as follows :

$$\begin{aligned}
 d_1 &= - \sum_{k=1}^n \frac{P_k}{M_T} + \sum_{k=1}^{n-1} \sum_{j=k+1}^{n-1} F_{kj} \\
 d_2 &= - \sum_{k=1}^n \sum_{j=k+1}^n H_{kj} \theta_{kj}^{(1)} \\
 d_3 &= - \sum_{k=1}^{n-1} \sum_{j=k+1}^n [F_{kj} \theta_{kj}^{(1)2} + H_{kj} \theta_{kj}^{(2)}]
 \end{aligned} \tag{A2}$$

Where

E. 44 Dr. Attia A.A.

$$\begin{aligned}
 A_{ij} &= [ C_{ij} \sin \theta_{ij} + D_{ij} \cos \theta_{ij} ] / M_i \\
 B_{ij} &= [ -C_{ij} \cos \theta_{ij} + D_{ij} \sin \theta_{ij} ] / M_i \\
 F_{ij} &= 2 D_{ij} \cos \theta_{ij} / M_T \\
 H_{ij} &= 2 D_{ij} \sin \theta_{ij} / M_T
 \end{aligned}
 \tag{A3}$$

APPENDIX 2

For system #2 shown in Fig.5 :  
the line parameters in per unit based on 100 MVA, 220 KV are as follows

NC	SB	EB	RL	XL	YC
2	9	10	.0004	.0031	.0360
2	9	11	.0014	.0103	.1188
2	9	15	.0032	.0235	.2708
2	10	11	.0014	.0103	.1188
2	11	12	.0024	.0177	.2045
2	11	22	.0043	.0312	.3601
2	12	13	.0007	.0053	.0612
2	13	14	.0051	.0259	.1580
1	13	20	.0162	.0809	.1246
1	13	21	.0111	.0561	.1170
2	14	17	.0023	.0172	.1981
2	15	16	.0013	.0094	.1080
2	16	17	.0032	.0234	.2701
2	17	18	.0020	.0099	.0606
2	18	19	.0036	.0181	.1106
2	19	20	.0039	.0200	.1216
2	21	22	.0064	.0324	.1975
2	22	23	.0032	.0162	.0987

The generator internal voltages and their initial angles are given in p.u by

$$\begin{aligned}
 E_1 \angle \delta_{10} &= 1.076 \angle 9.513^\circ \\
 E_2 \angle \delta_{20} &= 1.147 \angle 9.140^\circ \\
 E_3 \angle \delta_{30} &= 1.202 \angle 11.778^\circ \\
 E_4 \angle \delta_{40} &= 1.276 \angle 18.480^\circ
 \end{aligned}$$

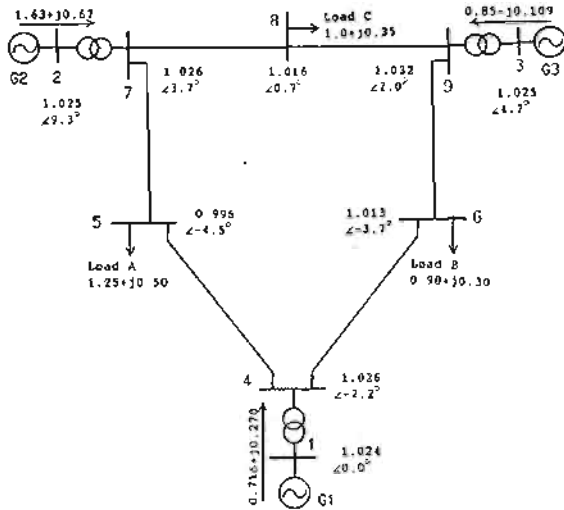


Fig.1 Power System # 1

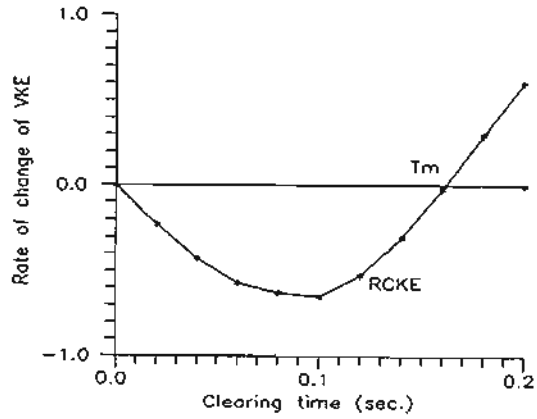


Fig.3 Curve of RCKE vis clearing time.

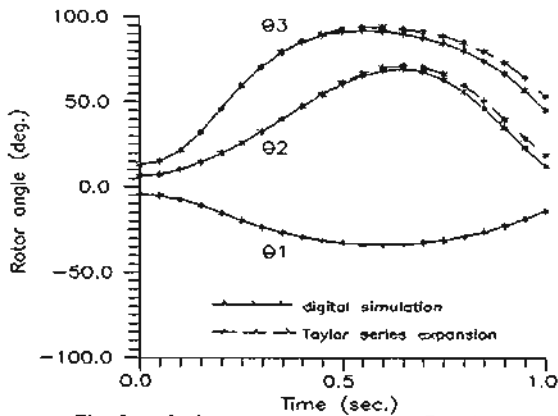


Fig.2 Swing curves for a 3-phase fault (160 ms) on busbar 7 of system #1

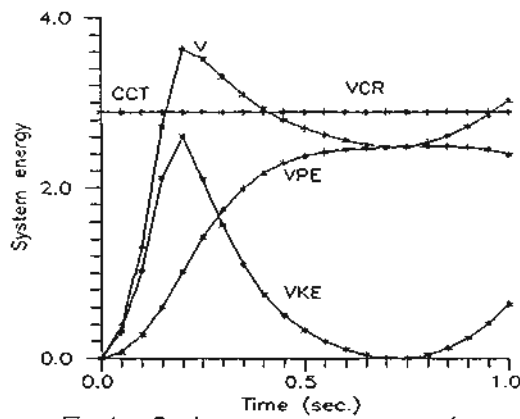


Fig.4 System energy curves for 3-phase fault (160 ms) at busbar 7 of system #1

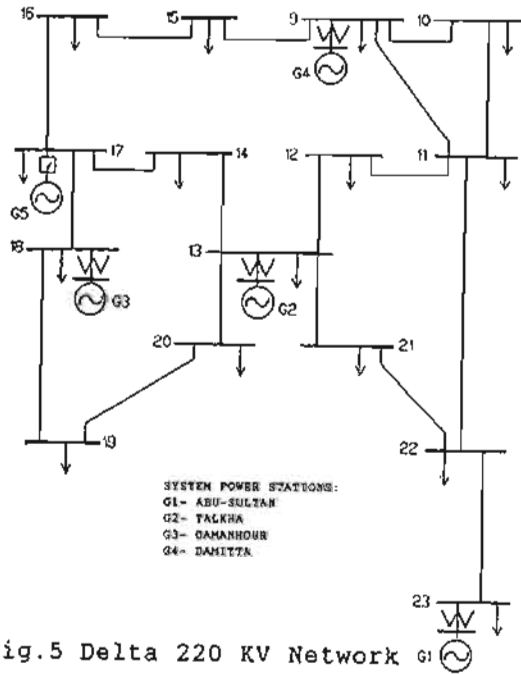


Fig.5 Delta 220 KV Network G1

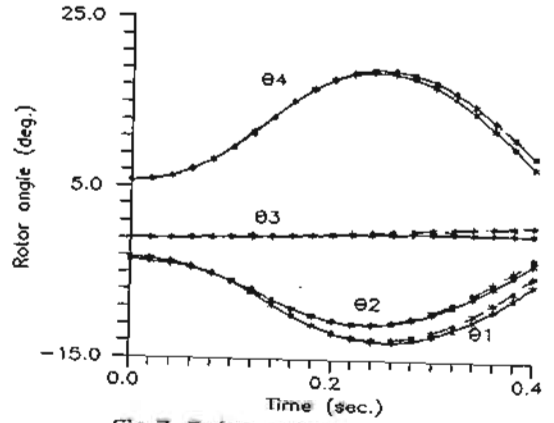


Fig.7 Swing curves as in Fig.5  
 — digital simulation  
 - - Taylor series expansion

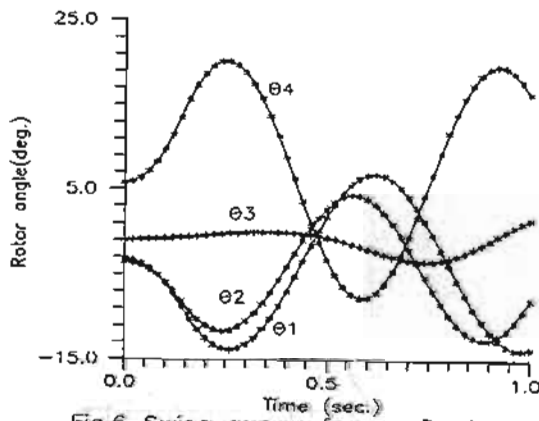


Fig.6 Swing curves for a 3-phase fault (120 ms) on busbar 9 of system #2

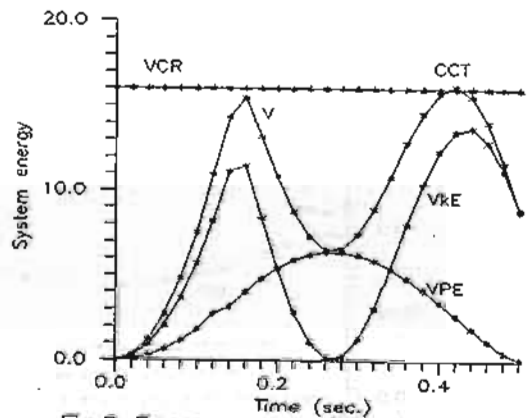


Fig.8 Energy curves for a 3-phase fault (120 ms) on busbar 9 of system #2





